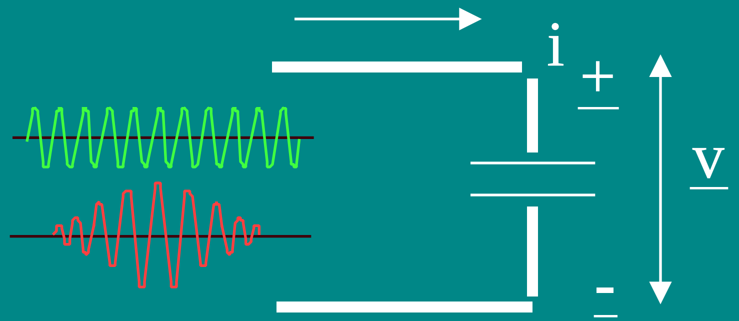
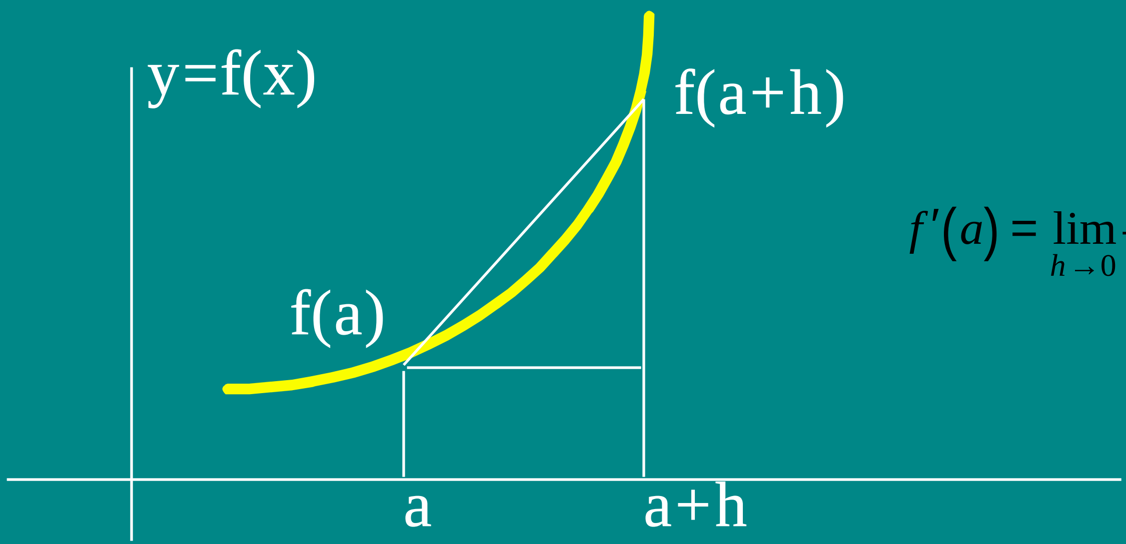
DERIVATIVES AND DIFFERENTIAL EQUATIONS



$$i = C\frac{dv}{dt}$$

WHAT IS A DERIVATIVE?

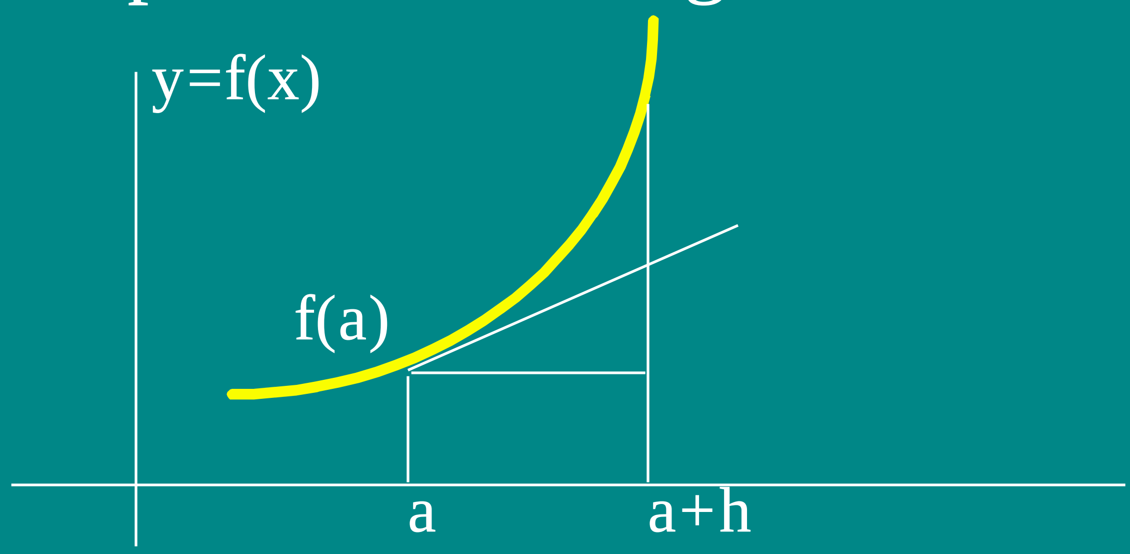
- Derivative of f(x) at x=a is given by



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

SIGNIFICANCE OF DERIVATIVE

- In the limit, derivative at $x=a$ is equal to the slope m of the tangent line at a.

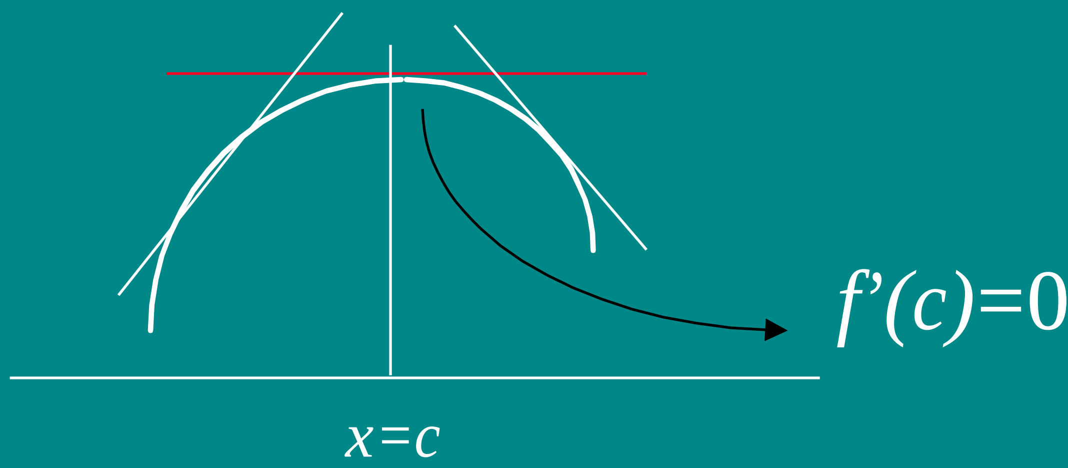


APPLICATIONS OF DERIVATIVE

- The most widely used application of derivative is in finding the *extremum* points of a function.
- If a function has a local extremum at a number c then either $f'(c)=0$ or $f'(c)$ doesn't exist

CRITICAL NUMBERS

- A number *c* in the domain of a function *f* is a critical number of *f* if either $f'(c)=0$ or $f'(c)$ does not exist



DERIVATIVES IN MATLAB:*diff*

- MATLAB computes two differentials, *dy* and *dx*, using *diff*.
- If x and y are input array of numbers

dy=*diff*(y)

dx=*diff*(x)

- Then,

yprime=*diff*(y)./ *diff*(x)

ILLUSTRATING *diff*

- Let *y*=[10 25 30 50 10] for *x*=[1 2 3 4 5]
- Then

diff(y)=[15 5 20 -40] ← One fewer component

diff(x)=[1 1 1 1 1]

- Dividing term-by-term

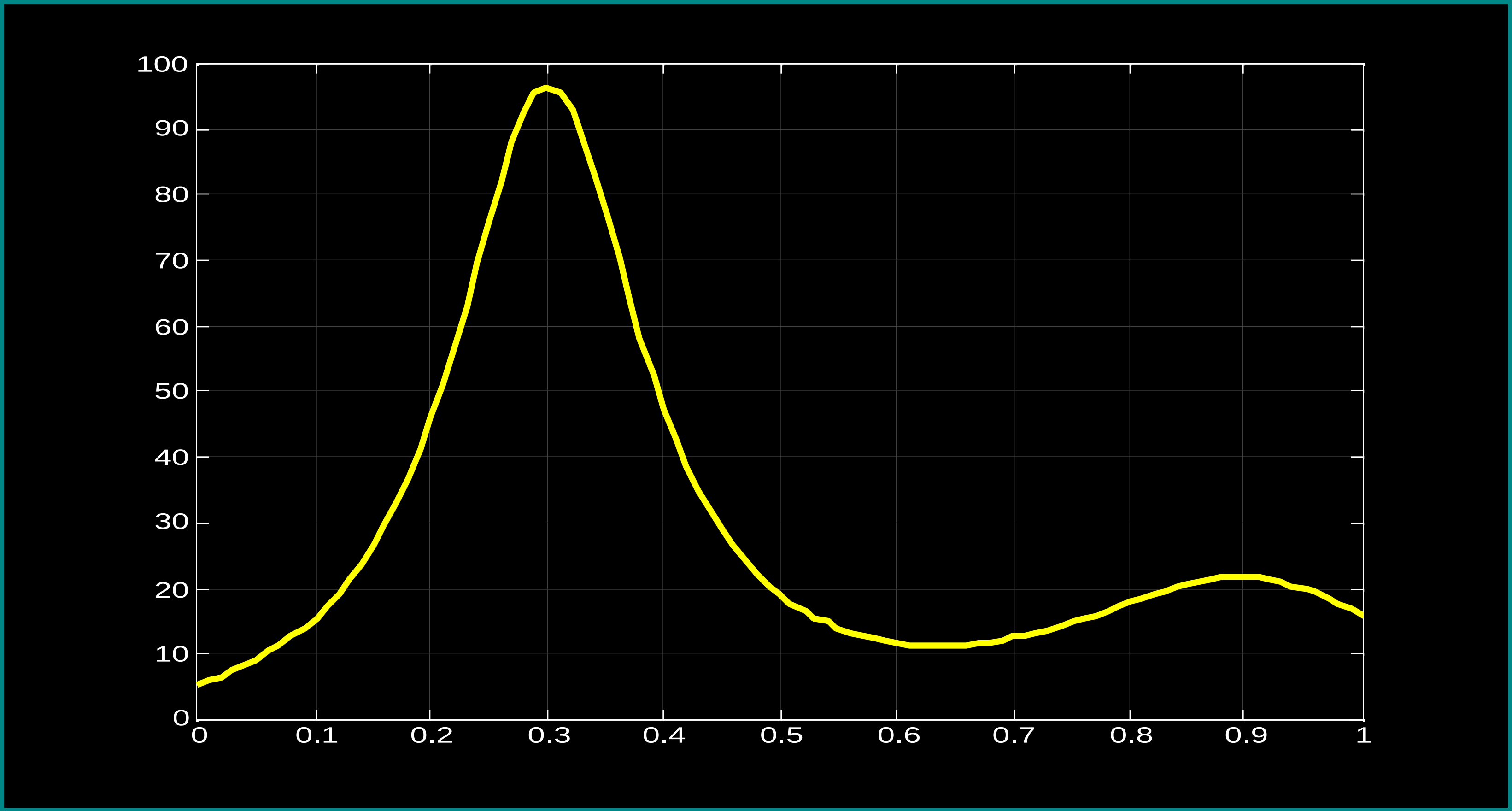
yprime=[15 5 20 -40]

WORKING WITH *humps*

- *humps* is a built-in MATLAB function, like peaks

$$y = \frac{1}{\left[(x-.3)^2 + 0.01\right]} + \frac{1}{\left[(x-.9)^2 + .04\right]} - 6$$

HOW IT LOOKS



DERIVATIVE OF *humps*

- First, we must know how *humps* was created in order to know *dx*

x=0:0.01:1;

y=*humps*(x);

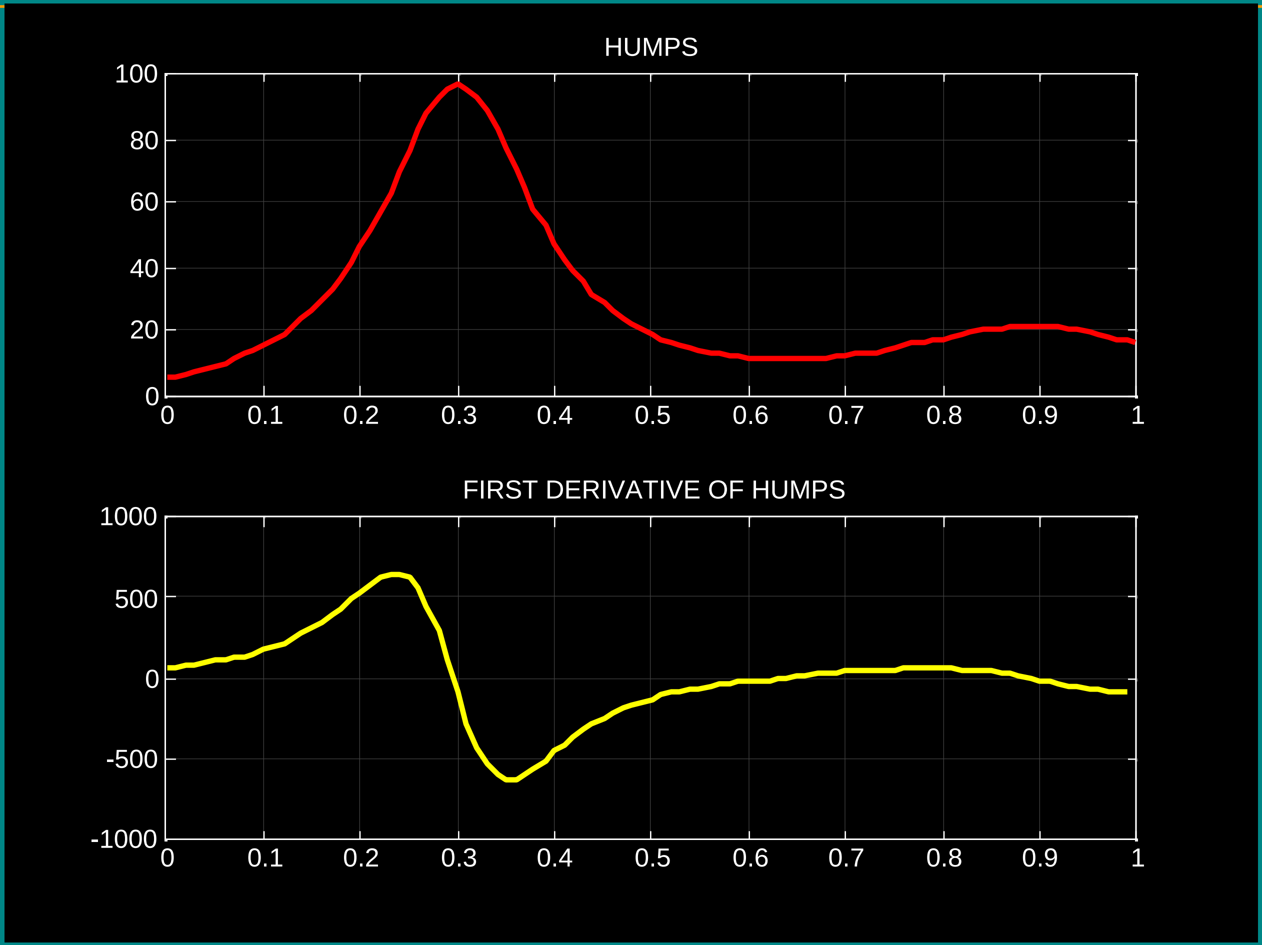
- Now use

dy=*diff*(y);

dx=*diff*(x);

yprime=dy./dx;

PLOT OF *dy/dx*



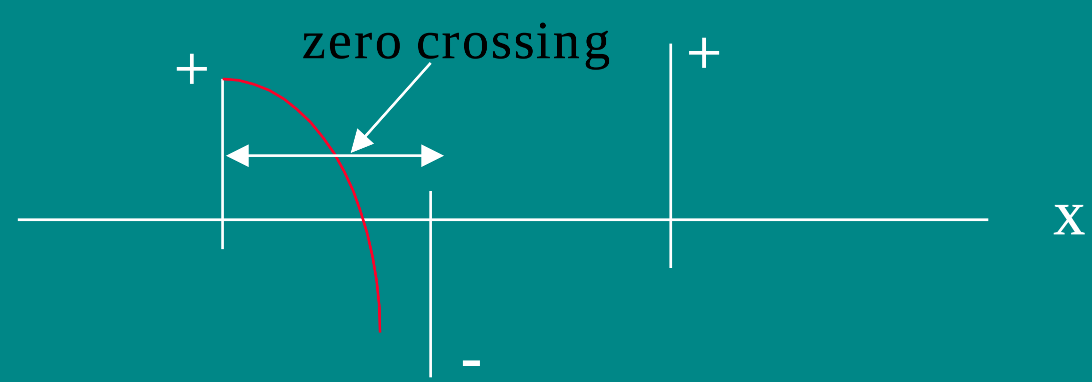
HUMPS
100
80
60
40
20
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
FIRST DERIVATIVE OF HUMPS
1000
500
0
-500
-1000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FINDING *critical points:* MATH vs. APPLICATION

- We should look for instances where y'=0.
- This is where the difference between textbook methods and real world shows up.
- To see for yourself, look for instances where yprime=0.

WHERE ARE THEY?

- Since there are no points where *y'* is exactly zero, we should look for points of *transition* from positive to negative, i.e. sign changes



FINDING ZERO CROSSINGS

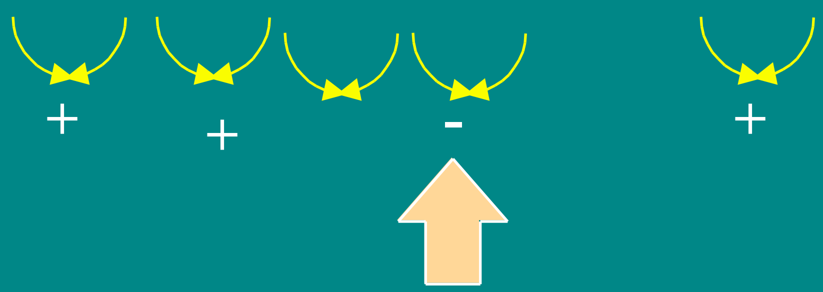
- There are two ways to find where a function crosses zero;

1. Numerical
2. Algebraic(polynomial fitting)

ZC USING ARRAY OPERATION

- To find where a sign transition takes place, multiply two consecutive numbers in the derivative array

[10 15 16 9 -4 -3 -4 -9]



MATLAB WAY

- Let *x* be an *n* element array. To generate the zero crossing array

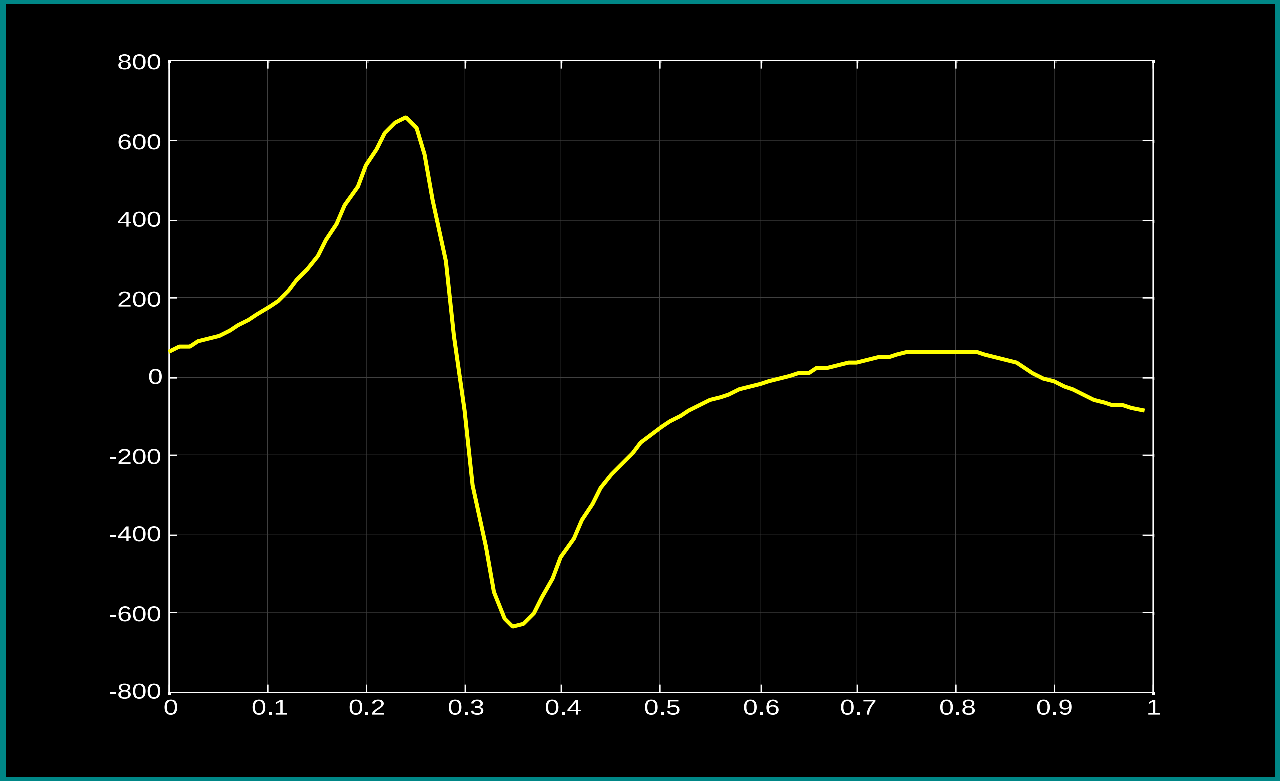
```
y=x(1:n-1).*x(2:n)
```

- Algebraically

$$y=[x_1x_2 \;\; x_2x_3 ,..., x_{n-1}x_n]$$

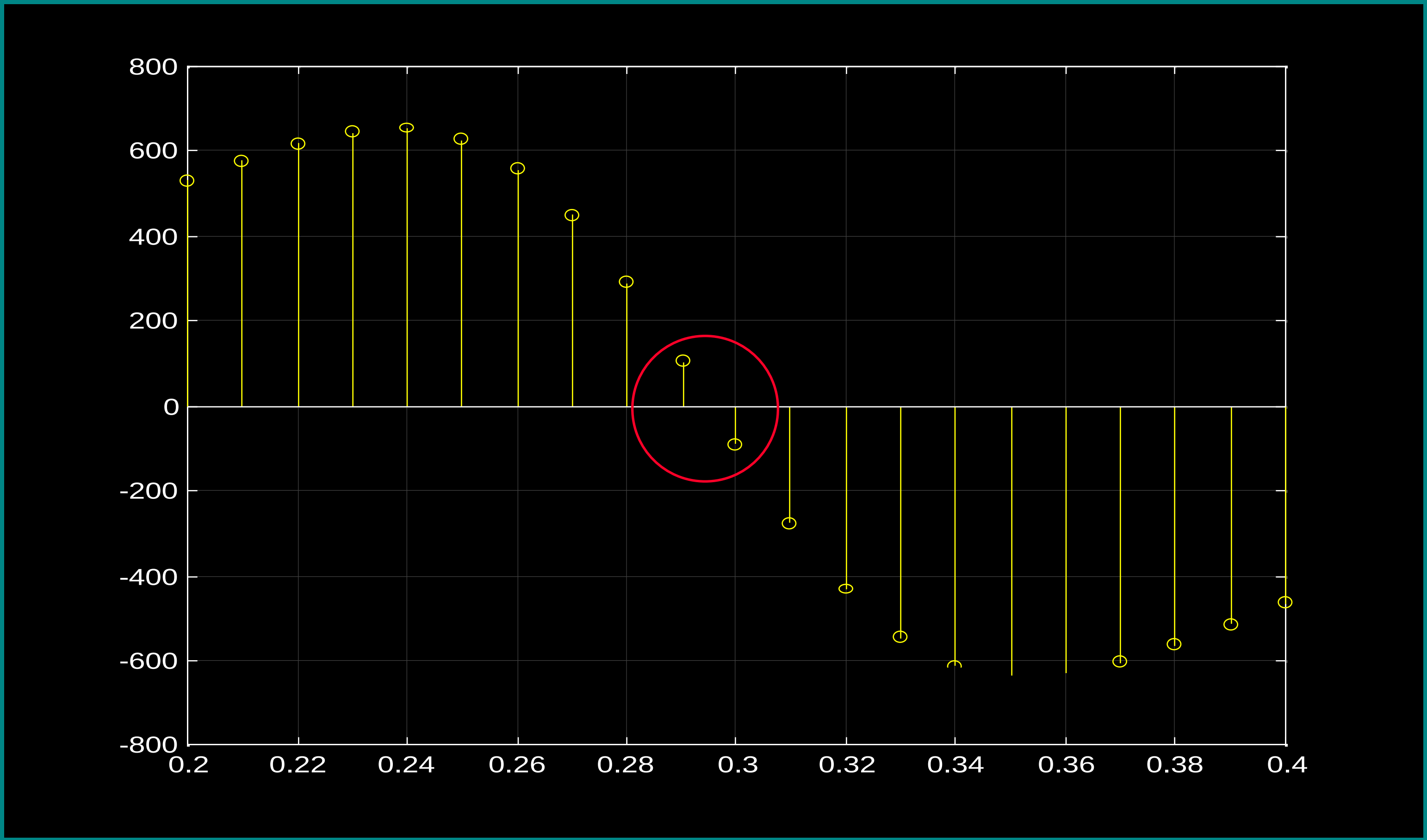
LOCATING SIGN CHANGES

- Just look for instances of negative sign in the zero crossing array



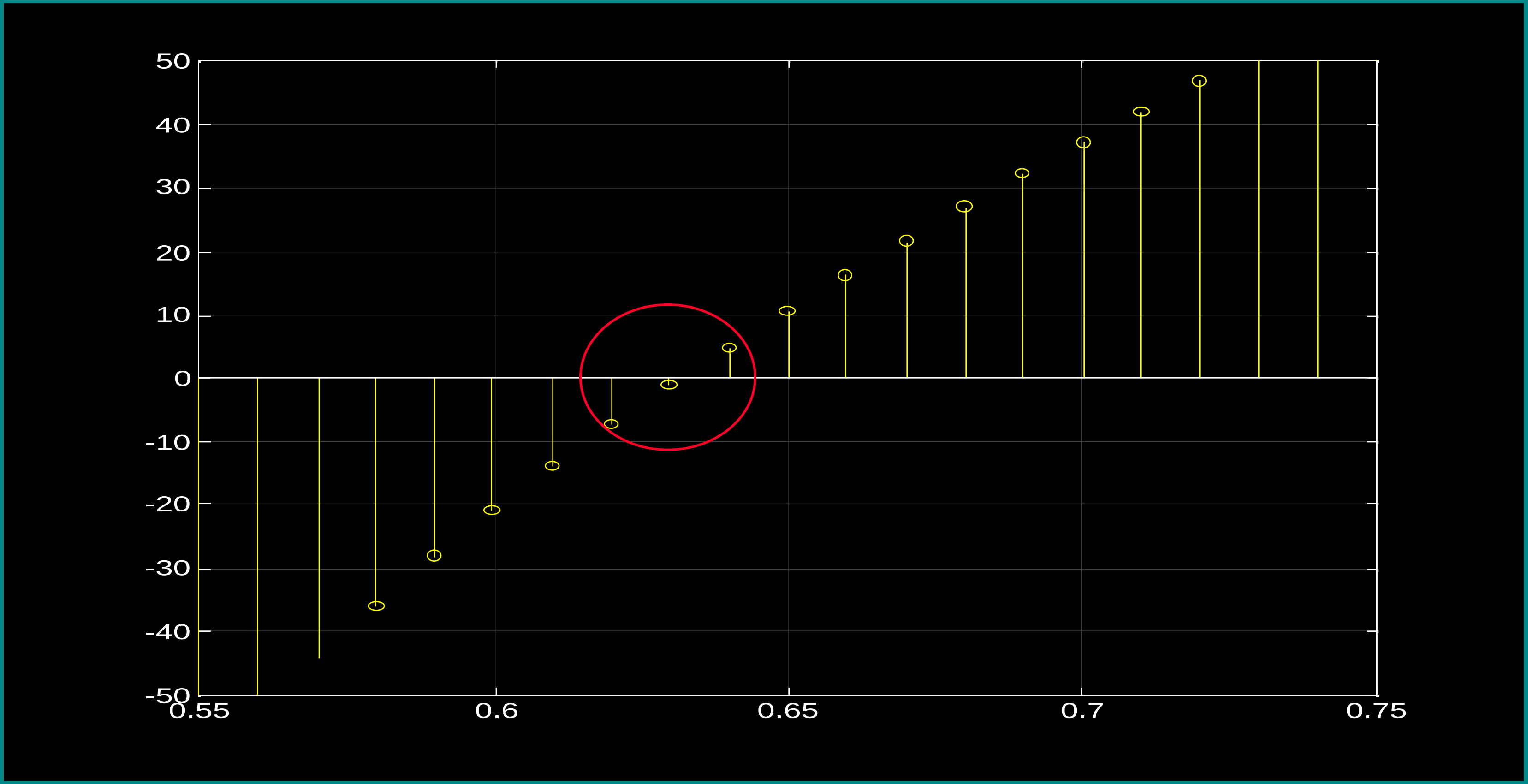
$c_1=0.29$
$c_2=0.63$
$c_3=0.88$

CLOSER LOOK

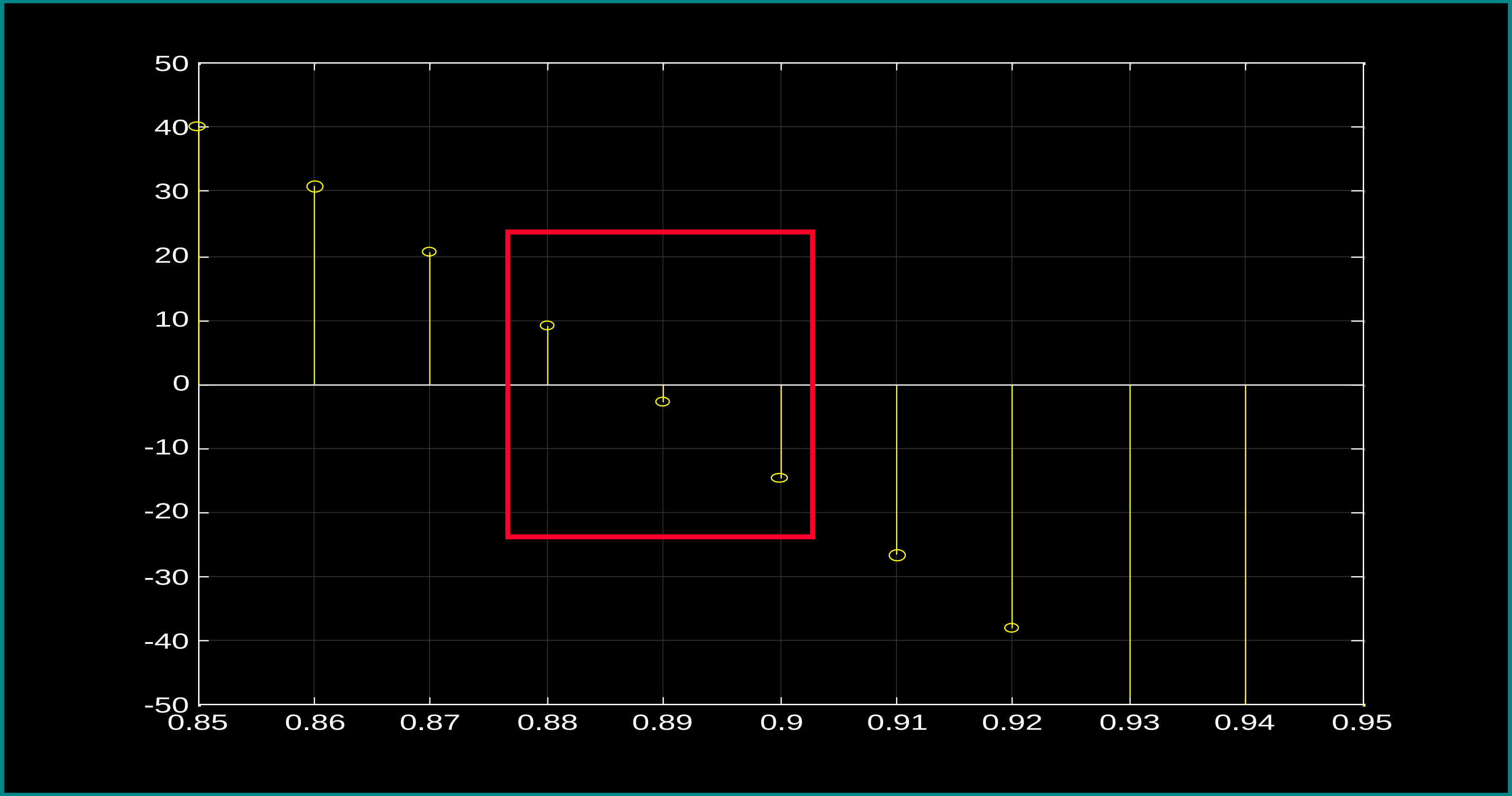


800
600
400
200
0
-200
-400
-600
-800
0.2
0.22
0.24
0.26
0.28
0.3
0.32
0.34
0.36
0.38
0.4

SECOND ZC



THIRD ZC



50
40
30
20
10
0
-10
-20
-30
-40
-50
0.85
0.86
0.87
0.88
0.89
0.9
0.91
0.92
0.93
0.94
0.95

DIFFERENTIAL EQUATIONS

- A first order differential equation is of the form

$$y' = dy/dx = g(x,y)$$

- We are looking for a *function* y such that when differentiated it gives us $g(x,y)$

FEW EXAMPLES

- Find *y* that such that

$$y' = 3x^2$$

$$y' = y$$

$$y' = (2x)\cos^2 y$$

- Answers:

$$y = x^3$$

$$y = \exp(x)$$

$$y = ???$$

SOLVING ODE's USING MATLAB

- MATLAB solves ODE in two ways: 1):numerical and 2):symbolic
- For numerical solution use

  [x,y]=ode23('function',a,b,initial)

EXPLAINING ode23

- Here are the elements of

[x,y]=ode23('function',a,b,initial)

function - this is $g(x,y)$ in $y'=g(x,y)$. Must be written as a separate MATLAB function

a - left point of the interval
b - right point of the interval
initial - initial condition, i.e. y(a)

USING ode23

- Let's solve $y' = 3x^2$
- First write a function as follows

function yprime=anything(x,y)

*yprime=3*x.^2;*

- Then call

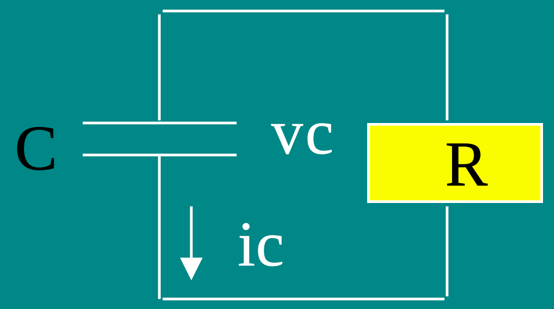
[x,y]=ode23('anything',2,4,0.5)

- *y* is the solution. Plot *x* vs.*y*

RC CIRCUIT

- Equation describing a source-free RC circuit is

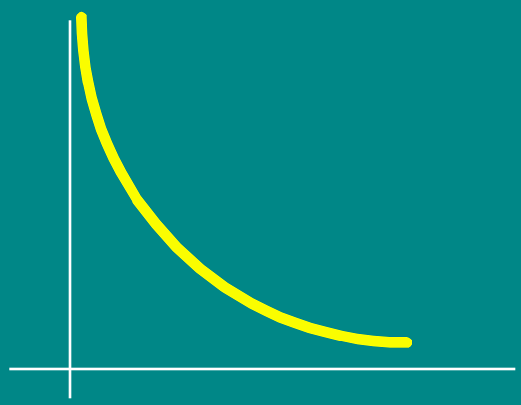
$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = 0$$



SOLVING FOR VOLTAGE

- The analytical solution is

$$v_c(t) = V_o e^{-t/RC}$$



- where Vo is the initial voltage across the capacitor

MATLAB's WAY

- To use ode23, we need to cast the problem in the form of $y' = g(x,y)$
- Here, y is v_c. . Therefore,

$$v_c' = -(1/RC)v_c$$

- With RC=10^-3, write a function like

function vcprime=rc(t,vc)

*vcprime=-(10^3)*vc*

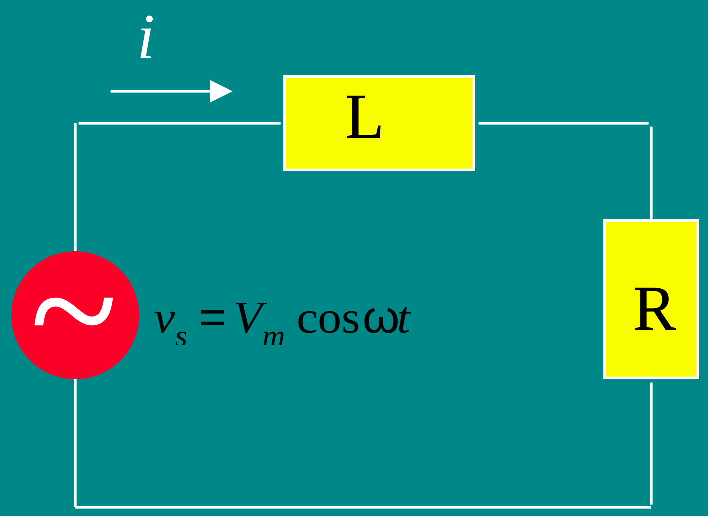
SOLVING FOR v_c

- Use *ode23* as follows

```
[t,vc]=ode23('rc',0,1,2)
```

CIRCUIT WITH FORCED RESPONSE

- Take the following RL circuit(p.497)



$$L\frac{di}{dt} + Ri = V_m \cos \omega t$$

WHAT IS i ?

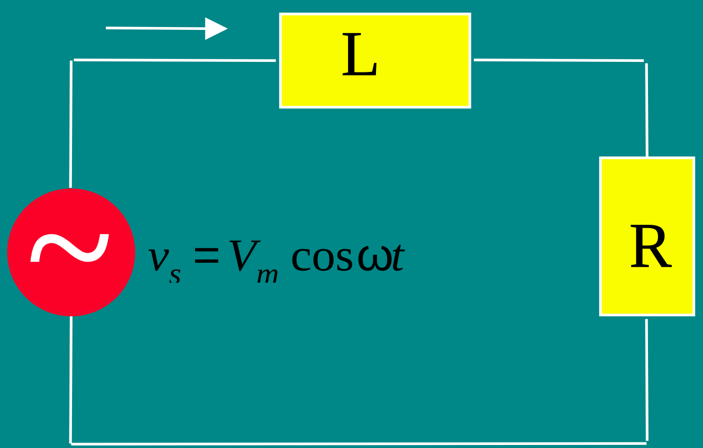
- From circuits analysis we know

$$i = \frac{V_m}{Z}\cos(\omega t - \beta)$$

where

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\beta = \tan^{-1}\frac{\omega L}{R}$$



SOME NUMBERS

- Let R=2 ohm, L=1 H and v_s=10cos3t
- Here is what we have

Z=sqrt(9+4)=sqrt(13)

V_m=10

β=56.3°

$$i = \frac{10}{\sqrt{13}}\cos\left(3t - 56.3\right)$$

EXERCISE- VERIFY WITH MATLAB

- Use *ode23* to solve for current. Match the two results, graphically, to see if the amplitude, phase and frequencies match the theoretical result