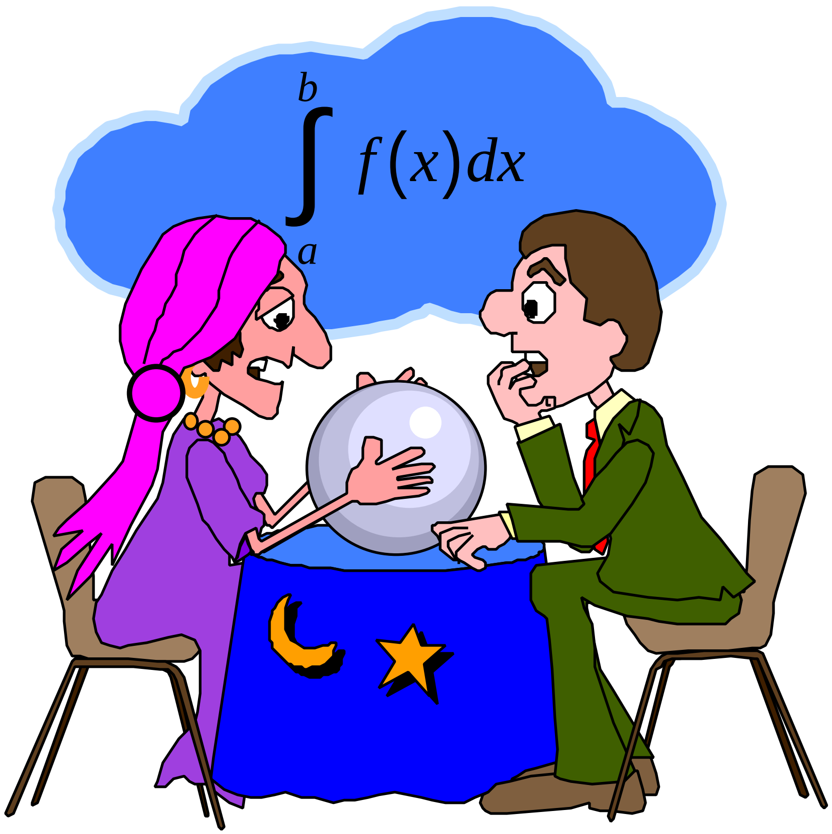
WEEK 15



EVALUATING DEFINITE

- MATLAB can evaluate definite

$$\int_a^b f(x)dx$$

- This is provided that the integrand f(x) be available as a ***function***, not an array of numbers

HOW DOES MATLAB DO IT?

- The primary function for evaluating definite integrals is quad8
- ***quad8*** has the following syntax
  - **q=quad8(‘function’,a,b)**
- This is equivalent to the expression

$$\int_a^b (function)dx$$

BUILT-IN MATLAB FUNCTIONS

- Evaluate the following

$$\int_0^{3\pi/2} \cos(x)dx$$

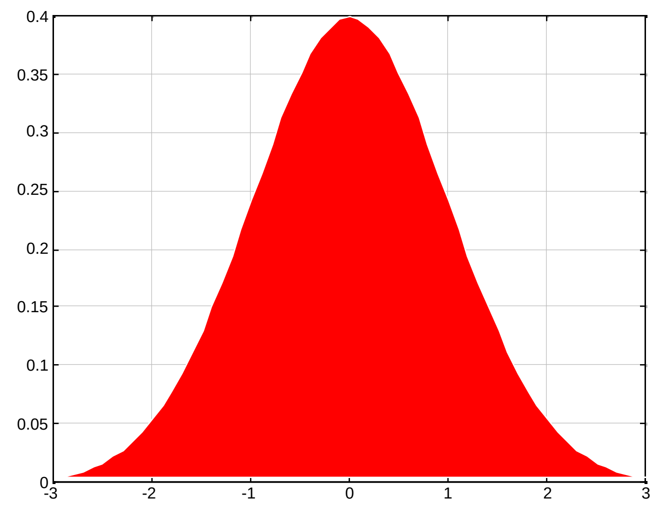
- Since cosine is a built-in MATLAB function;

y=quad8('cos',0,3*pi/2)

USER-DEFINED FUNCTION

- You can integrate functions that are not part of MATLAB library.
- For example, write a function of your own, gauss,

$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

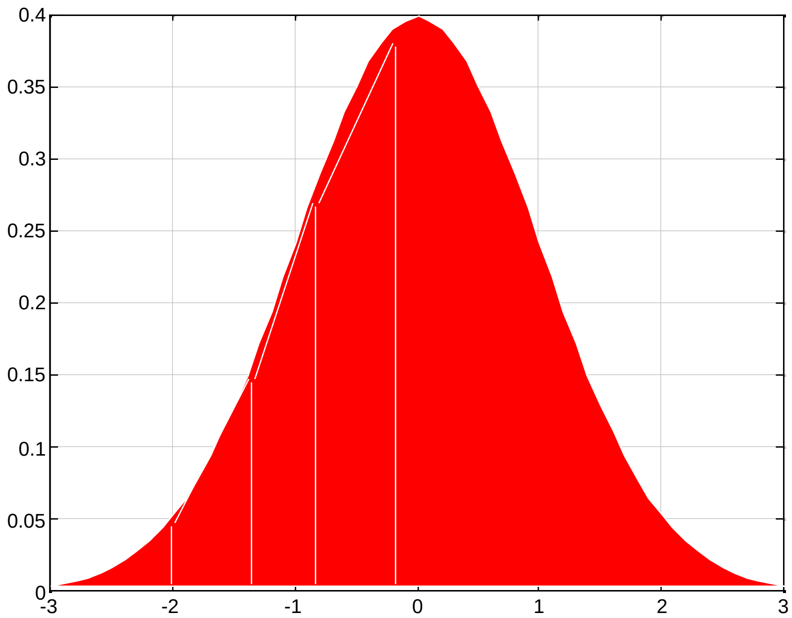


USING *gauss* IN QUAD8

- Now let's find the area under Gaussian curve within various intervals
  - **area=quad8('gauss',-4,4)**
  - **area=quad8('gauss',0,4)**
  - **-area=quad8('gauss',-0.5,1)**

APPROXIMATION TO INTEGRALS - *trapz*

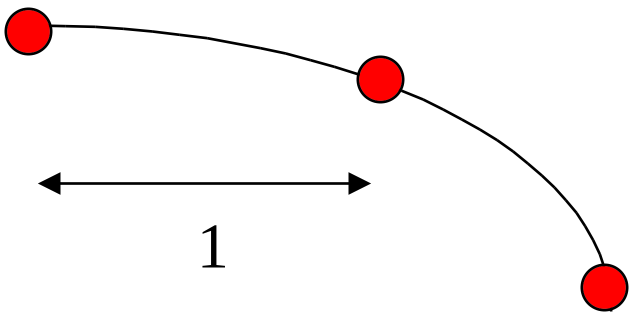
- Function ***trapz*** uses areas of trapezoids to approximate the area under the curve



area=trapz(x,y)

Sample Spacing

trapz assumes unit spacing between samples



If that is not true, the output of trapz must be scaled by the actual spacing, e.g. 0.1

IN-CLASS EXERCISE

- Energy of a signal

$$E = \int_{T_1}^{T_2} s^2(t)dt$$

- Using ***quad8,*** find the energy of a gaussian pulse in the range (-1,1)

EXTENSION OF 1D INTEGRALS

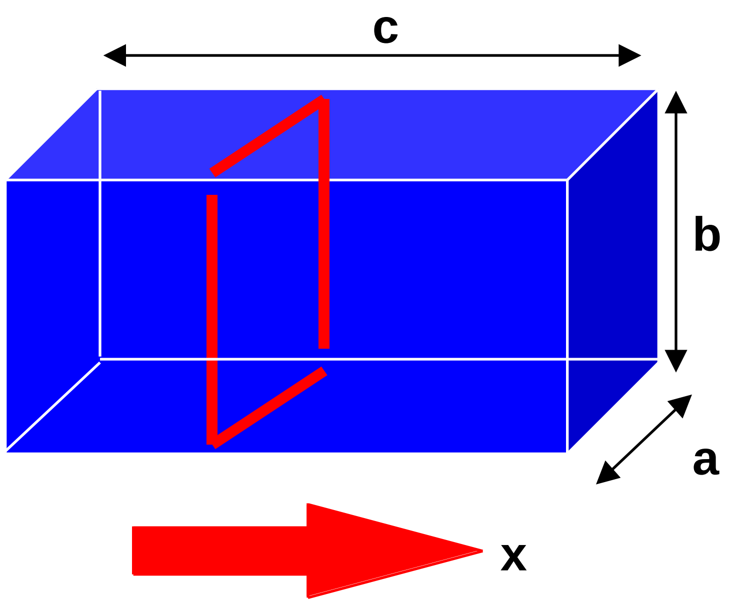
- 1-D integral can geometrically be interpreted as an area.
- It is possible to evaluate volumes, not by multidimensional integrals as is generally done , but as 1-D integrals.

DEFINING VOLUMES

- There are a number of ways a 3D shape can be generated
  - **Sweeping a Cross Section**
  - **The Disc Method**
  - **The Washer Method**
  - **The Shell Method**

CROSS-SECTIONAL METHOD

- Imagine sweeping a 1D shape, of varying cross sections A(x), along a path. This action will generate a swept volume.



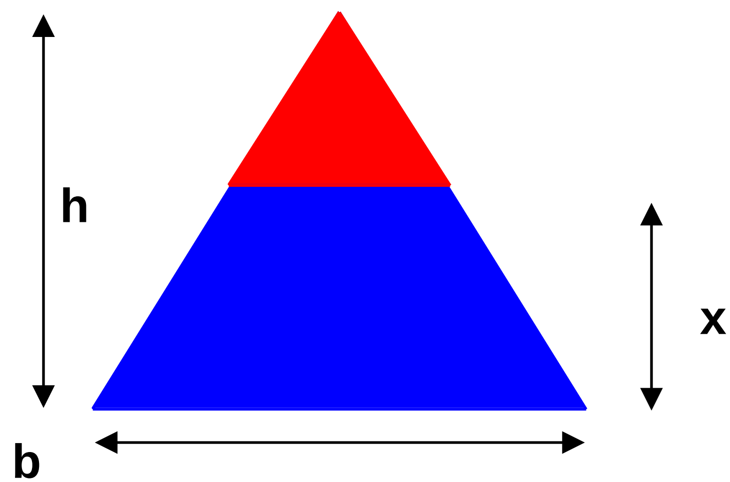
$$V = \int_a^b A(x)dx$$

$$= \int_0^c (a \times b)dx = abc$$

VOLUME OF A PYRAMID

- In problems like this you must first do two things
  - **write a function for the cross section as a function of x**
  - **determine the lower and upper limit of the sweep**

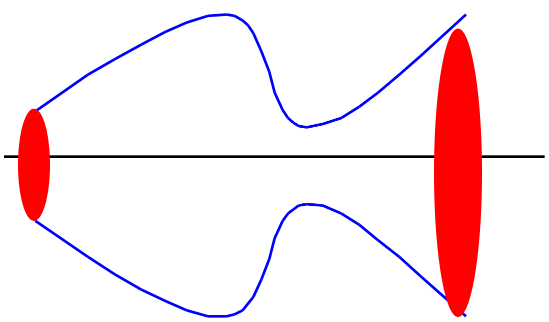
$$A(x) = \left[\frac{b}{h}(h-x)\right]^2$$



THE DISC METHOD

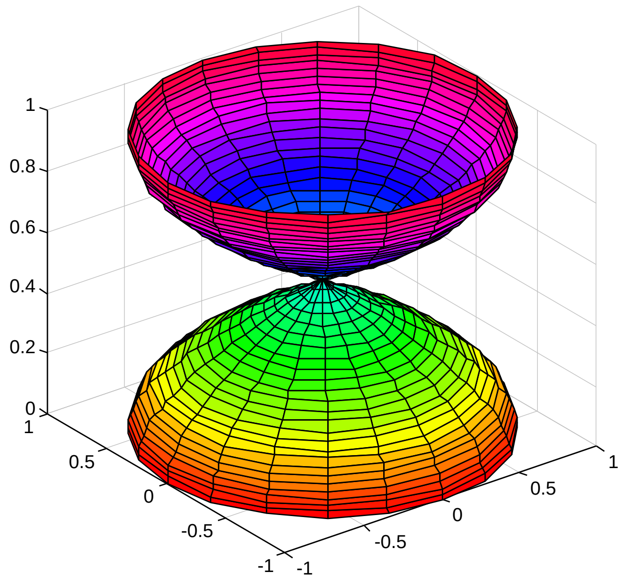
- Take a 1D curve f(x) and revolve it around the x-axis. This is a volume of revolution:
  - **semi-circle---> sphere**
  - **triangle --> cone**
- Every cross section is a circle. The radius of the circle at x_o is $f(x_o)$.

$$V = \int_a^b \pi [f(x)]^2 dx$$



REVOLVING A SINUSOID

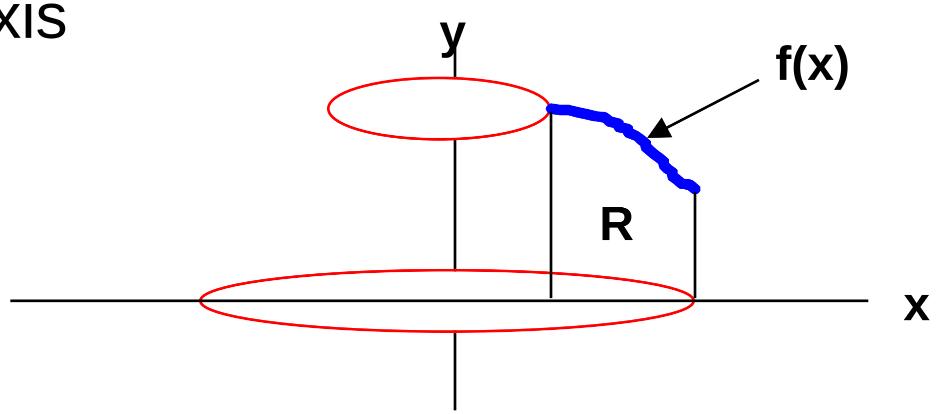
- Take one period of $1+\cos(2\pi x)$ and revolve it around the x-axis. Plot the shape then find the volume of the revolution



THE SHELL METHOD

- Define a function f(x) in a<x<b. Revolve R around the y-axis

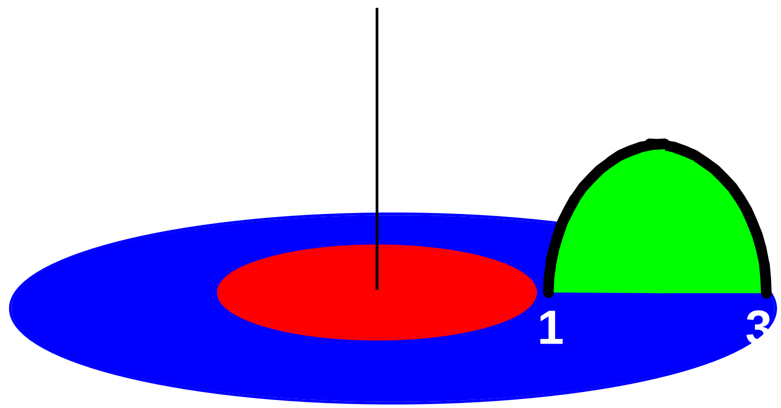
$$V = \int_a^b 2\pi x f(x)\,dx$$



- Examples:
  - **revolve a rectangle --> cylinder with a thickness**
  - **revolve a circle --> torus/donut**

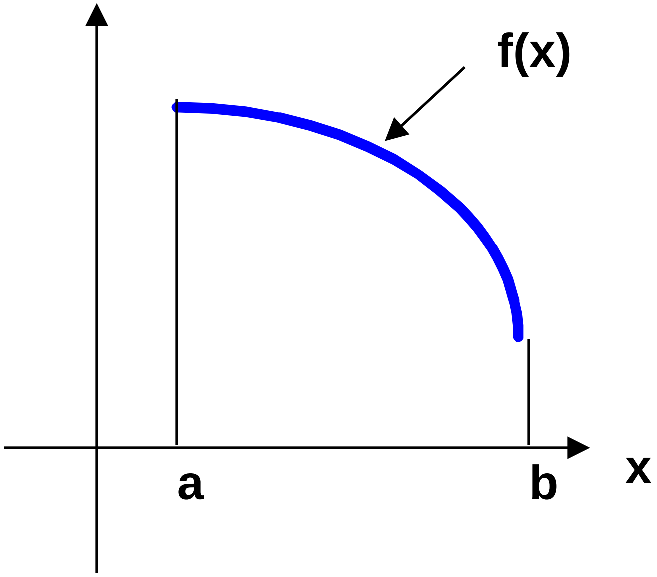
EXAMPLE

- Let $f(x)=1-(x-2)^2$ for $1<x<3$. Revolve this around the y-axis and find its volume



ARC LENGTH

- Another important application of integrals is finding arc lengths



$$L = \int_a^b \sqrt{1 + \left[f'(x) \right]^2} dx$$

PARAMETRIC CURVES

- It is frequently easier to work with a parametric representation of a curve,i.e.

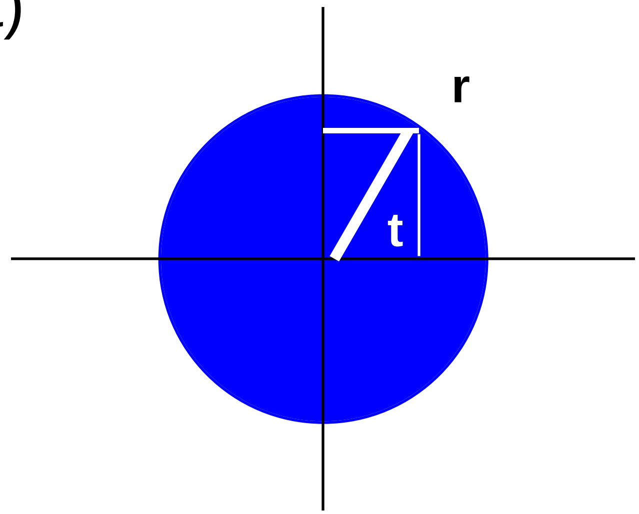
$$x=f(t)$$

$$y=g(t)$$

- For example, a circle

$$x(t)=r\cos(t)$$

$$y(t)=r\sin(t)$$



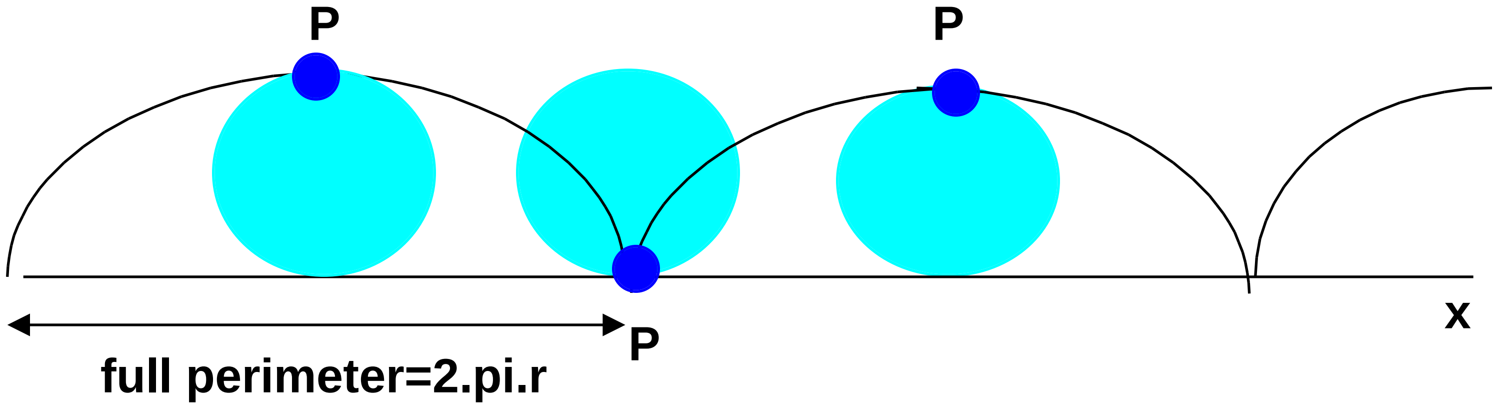
LENGTH OF PARAMTERIC CURVES

- Using derivatives of f(t) and g(t)

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt$$

CYCLOID

- Path length traversed by a point on a wheel is of interest



LENGTH OF A CYCLOID

- The parametric equation of a cycloid with r=1 is given by

$$x=t-\sin(2.pi.t)$$

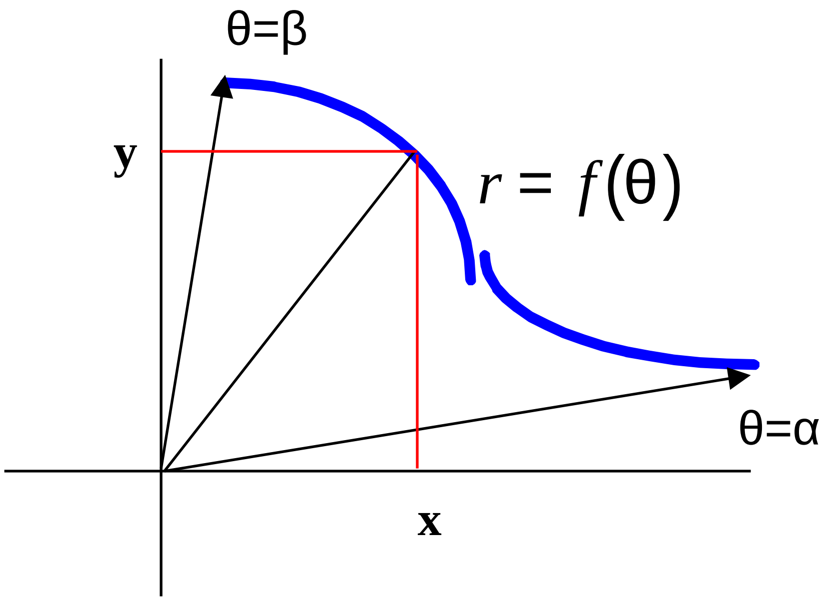
$$y=1-\cos(2.pi.t)$$

- First, plot it for $0< t <2$.
- Then find its length for one cycle and compare it with the horizontal distance

INTEGRALS IN POLAR COORDINATES

- A curve can be represented in polar coordinates by $$r = f(\theta)$$
- Equivalently

$$\begin{cases} x = f(\theta)\cos(\theta) \\ y = f(\theta)\sin(\theta) \end{cases}$$



CURVE LENGTH

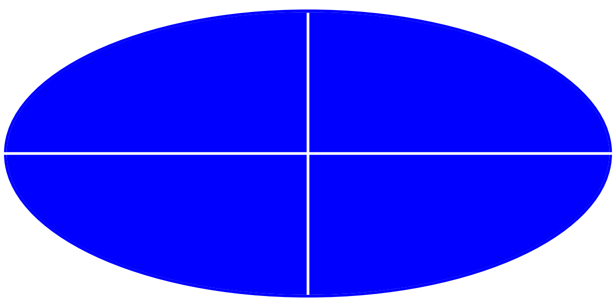
- The length of a curve represented in polar coordinates is given by

$$L = \int_{\alpha}^{\beta} \sqrt{[f'(\theta)]^2 + [f(\theta)]^2}\, d\theta$$

PERIMETER OF AN ELLIPSE

- Find the perimeter of an ellipse given by

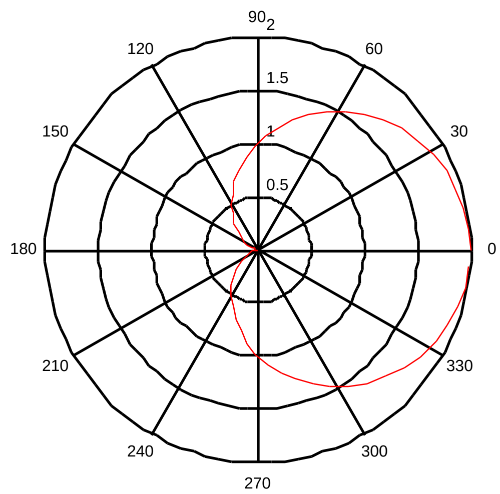
$$r = \frac{6}{3 + 2\cos\theta}$$

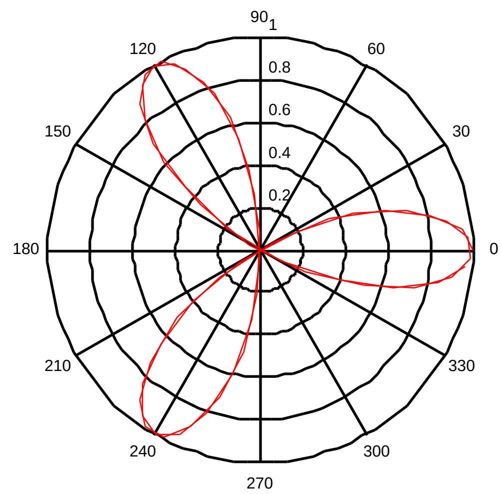


$$f'(\theta) = \frac{-12\sin\theta}{(3 + 2\cos\theta)^2}$$

cardioid, 3-leaved rose

- ***cardioid*** is defined by $r=1+\cos\theta$.
- 3-leaved rose is given by $r=\cos 3\theta$.



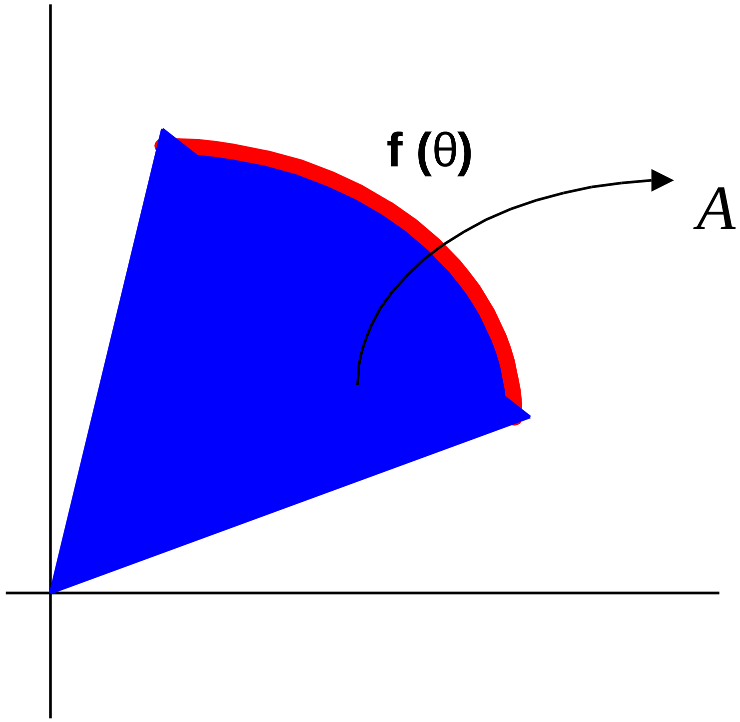


LENGTH OF A *cardioid*

- We need the derivative of $f(\theta)$
  - $f'(\theta) = -\sin(\theta)$
- Then,

$$L = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\sin^2\theta + (1+\cos(\theta))^2}\, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{2+2\cos\theta}\, d\theta$$

AREA IN POLAR COORDINATES



$$A = \int_{\alpha}^{\beta} \frac{1}{2}[f(\theta)]^2 d\theta$$

AREA OF AN ELLIPSE

- For the ellipse given by

$$r = \frac{6}{3 + 2\cos\theta}$$

find its area and verify

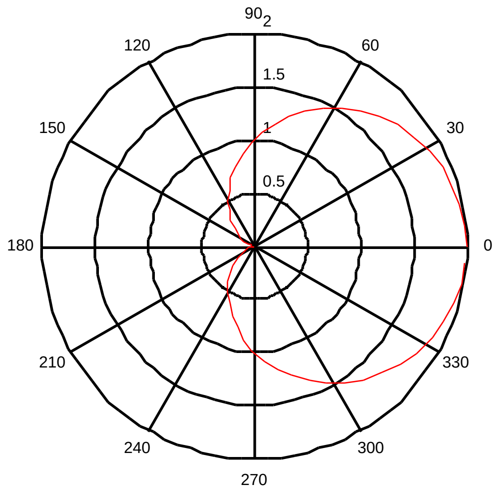
AREA OF A *cardioid*

- Here we have $f(\theta) = 1 + \cos\theta$
- Then

$$A = \int_0^{2\pi} \frac{1}{2}[f(\theta)]^2 d\theta = \int_0^{2\pi} \frac{1}{2}[1 + \cos\theta]^2 d\theta$$

HOMEWORK

- Find the length and area of a cardioid. Use the relevant equations for length and area given previously



HOMEWORK-2

- Find the energy of the bond clip using ***trapz***. This routine assumes unit sample spacing. However, bond is sampled at 8KHz. Take this into account.